Spatial Econometrics Methods using Stata

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General index

What is Spatial Econometrics?

Definitions of Spatial econometrics:

- Jean Paelinck introduced the term “Spatial Econometrics” in 1974 to designate: “a combination of economic theory, mathematical formalization and statistics with: role of spatial interdependence, importance of factors in other places, explicit modelling of space".
- Luc Anselin (1988) defined the Spatial Econometrics as: “econometric branch dealing with spatial interaction and spatial structure in cross-sectional models and data panel (separating spatial dependence and spatial heterogeneity)”.

Why is Spatial Econometrics important nowadays?

- Theory-driven
  - From individual decision to social-spatial interaction.
  - Common shocks.
  - Peer-effects, contextual effects, neighbourhood effects.
- Data-driven
  - Geo-referenced information.
- Technology
  - Geographical Information Systems.
  - Capability of statistical software.

Types of spatial data and Objective of workshop

- Types of spatial data:
  - Geostatistic data: continuous spatial field (noise surface, pollution surface).
  - Aerial (Lattice or Regional): discrete spatial data, fixed polygons or points (counties, provinces, countries).
  - Point pattern: location as a random event (crimes, accidents).

This workshop proposes a data-driven analysis using lattice data:

- Specifying the structure of spatial dependence.
- Testing for the presence of spatial effects.
- Estimating and interpreting models with spatial effects.

Example used: Impact of net migration on unemployment

- Hot topic in economics.
- Competitive theories:
  - Orthodox theory: net migration causes more unemployment (positive relationship).
  - New Economic Geography theory: net migration causes less unemployment (negative relationship).
- Definition of variables:
  - UNEMPLOYMENT RATE as the number of people unemployed as a percentage of the labour force.
  - NET MIGRATION RATE as the ratio of net migration during the year to the average population in that year. The value is expressed per 1,000 persons. Net migration is the difference between immigration into and emigration from the area during the year (net migration is therefore negative when the number of emigrants exceeds the number of immigrants).
- Level of analysis:
  - NUTS 2 (Europe 15), 164 regions from 2007 to 2012.

General Index

From shape to dta

- First we need a file of administrative areas: `http://www.diva-gis.org/`, `http://www.gadm.org/`.
- Georeferencing information (lattice data) usually is stored in a shapefile (a collection of files with a common filename with at least three connected files):

  .shp is the file that store geometric objects.
  .shx is an index file of geometric objects.
  .dbf is the database file, in dBASE format, and contains information of attributes of the objects.
- Shape files cannot be read directly in Stata.
- However, shp2dta command can import shapefiles and convert them in Stata format.

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sh2dta command

Syntax:

shp2dta using shp.filename, database(filename) coordinates(filename) [options]

Example:

shp2dta using nuts2_164, database(data_shp) coordinates(coord) genid(id) genc(c)

The shp2dta command generates two files:

- data_shp.dta: contains information from .dbf file, id, latitude (y_c) y longitude (x_c).
- coord.dta: contains geometric information from .shp file.

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Merging data sets

- The new database (data_shp.dta) does not contain information about economics variables.
- Using the index of geometric objects has been generated a excel file with variables from Eurostat: unemployment rate and net migration rate.
- Both dataset are easily jointed using POLY_ID as link variable:

```
import excel "C:\.\.\.\nuts2_164.xls", firstrow
save "C:\.\.\.\migr_unemp07_12.dta"
use data_shp, clear
merge 1:1 POLY_ID using migr_unemp, gen(union) force
```

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Visualizing in maps

A choropleth is a map in which each area is coloured with an intensity proportional to the value of a quantitative variable. Some classical maps:

- **Quantiles**: class breaks correspond to quantiles of the distribution of variable (each class includes approximately the same number of polygons).
- **Equal Intervals**: class breaks correspond to values that divide the distribution of variable attribute into k equal-width intervals.
- **Boxplot**: the distribution of variable attribute is divided into 6 classes defined as follows: $[min, p25 - 1.5 * iqr]$, $(p25 - 1.5 * iqr, p25]$, $(p25, p50]$,$(p50, p75]$, $(p75, p75 + 1.5 * iqr]$ and $(p75 + 1.5 * iqr, max]$, where iqr is the interquartile range.
- **Standard Deviates**: the distribution of variable attribute is divided into k classes $(2 \leq k \leq 9)$ whose width is defined as a fraction p of its standard deviation sd.

spmap command

Syntax:

spmap [attribute] [if] [in] using basemap [,basemap_options]

Details: basemap_options

polygon(polygon_suboptions)
line(line_suboptions)
point(point_suboptions)
diagram(diagram_suboptions)
arrow(arrow_suboptions)
label(label_suboptions)
scalebar(scalebar_suboptions)
graph_options]

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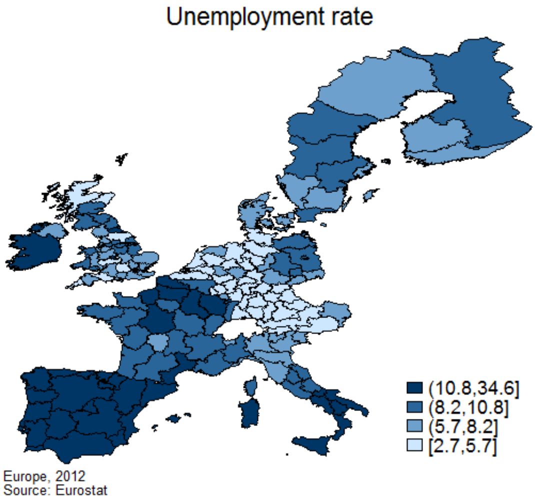
label(label_suboptions)
scalebar(scalebar_suboptions)
graph_options]

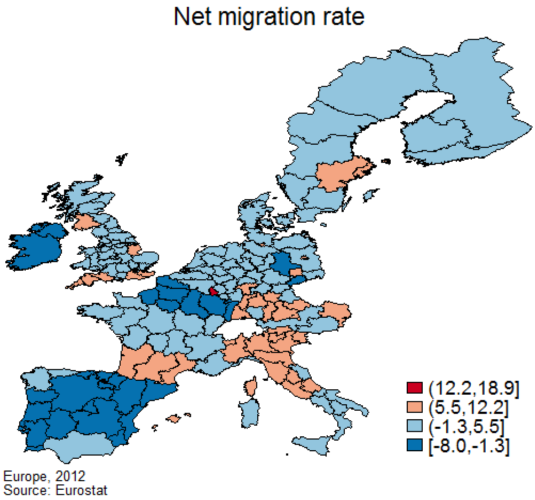
Quantile map

```
spmap U2012 using coord, id(id) clmethod(q) title("Unemployment rate")
legend(size(medium) position(5)) fcolor(Blues2) note("Europe, 2012" "Source:
Eurostat")
```



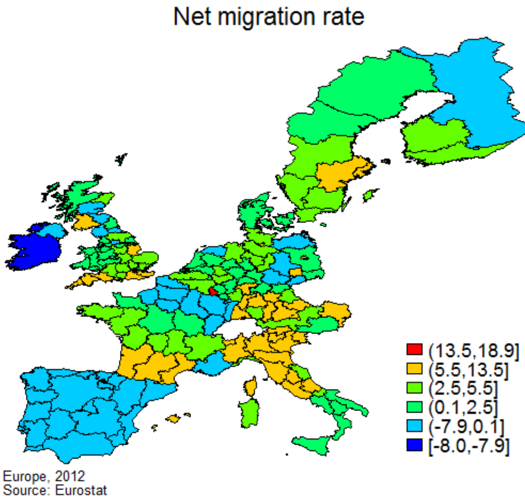
Equal intervals map

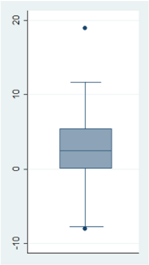
```
spmap NM2012 using coord, id(id) clmethod(e) title("Net migration rate")
legend(size(medium) position(5)) fcolor(BuRd) note("Europe, 2012" "Source:
Eurostat")
```



Box map

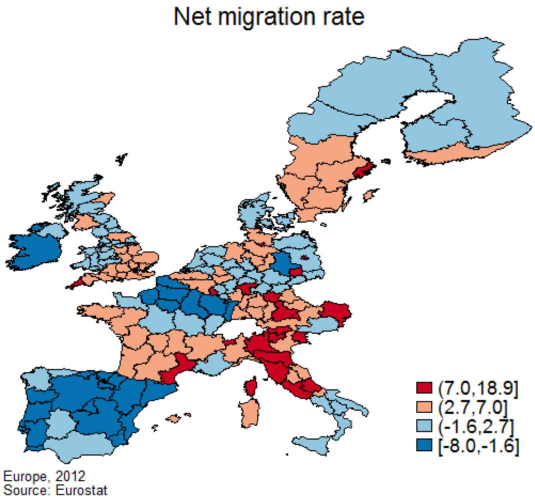
```
spmap U2012 using coord, id(id) clmethod(boxplot) title("Unemployment rate")
legend(size(medium) position(5)) fcolor(Heat) note("Europe, 2012" "Source:
Eurostat")
```





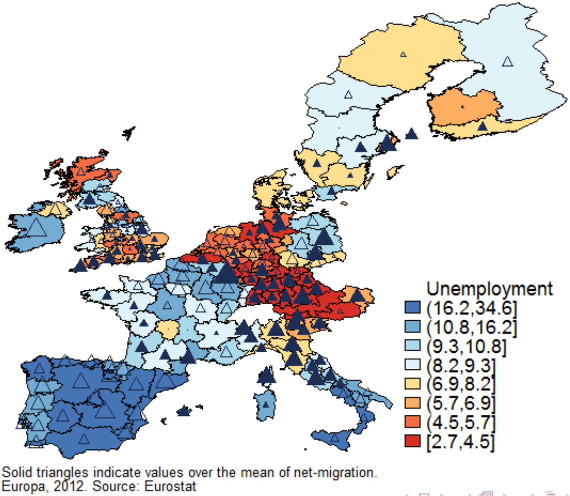
Deviation map

```
spmap NM2012 using coord, id(id) clmethod(s) title("Net migration rate")
legend(size(medium) position(5)) fcolor(BuRd) note("Europa, 2012" "Source:
Eurostat")
```

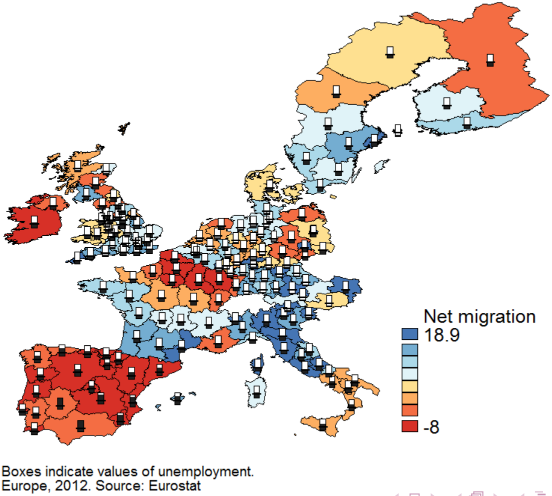


Combine map

```
spmap U2012 using coord, id(id) fcolor(RdYlBu) cln(8) point(data(migr_unemp_shp)
xcoord(x_c) ycoord(y_c) deviation(NM2012) sh(T) fcolor(dknavy) size(*0.3))
legend(size(medium) position(5)) legt(Unemployment) note(" " "Solid triangles
indicate values over the mean of net-migration." "Europa, 2012. Source: Eurostat")
```



```
spmap NM2012 using coord, id(id) fcolor(RdYlBu) cln(8) diagram(var(U2012)
xcoord(x_c) ycoord(y_c) fcolor(gs2) size(1.7)) legend(size(medium) position(5))
legstyle(3) legt(Net migration) note("Boxes indicate values of unemployment."
"Europe, 2012. Source: Eurostat")
```



General Index

Centrality of spatial W

- We show spatial concentration in previous maps, in a formal way:

$$\begin{bmatrix} y_i \\ y_j \\ y_k \end{bmatrix} = \begin{bmatrix} 0 & \alpha_{ij} & \alpha_{ik} \\ \alpha_{ji} & 0 & \alpha_{jk} \\ \alpha_{ki} & \alpha_{kj} & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_j \\ y_k \end{bmatrix} + \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}, (6) \quad (1)$$

$$y = Ay + u, \quad (2)$$

- Strategy of identification:

$$A = \begin{bmatrix} 0 & \alpha_{ij} & \alpha_{ik} \\ \alpha_{ji} & 0 & \alpha_{jk} \\ \alpha_{ki} & \alpha_{kj} & 0 \end{bmatrix} = \rho \begin{bmatrix} 0 & w_{ij} & w_{ik} \\ w_{ji} & 0 & w_{jk} \\ w_{ki} & w_{kj} & 0 \end{bmatrix} = \rho W.$$

  We transform a non-identified model in other that contains only one parameter: ρ.

- W captures 'who is the neighbour of whom': must be EXOGENOUS!

Criteria used to create W

Usually, the building of W is an ad-hoc procedure of the researcher. Common criteria are:

1. Geographical:
    - Distance functions:
        - inverse
        - inverse with threshold
    - Contiguity:
        - Rook
        - Queen
    - K nearest neighbours.
2. Socio-economic:
    - Similarity degree in economic dimensions (or social networks).
3. Combinations between both criteria.

Generating W using Stata

In Stata there are (at least) three commands to generate W:

- `spatwmat`:
  - Distance criterion.
  - Used for spatial univariate analysis.
  - Format file no compatible with `spmat`.
- `spwmatrix`:
  - Generate W using geographic criteria (no contiguity).
  - Generate W under socio-economic criteria.
  - Import, export and manipulate from GeoDa.
  - Compatible format file with `spatwmat`.
- `spmat`:
  - Generate W using geographic criteria (no under knn).
  - Import, export and read matrices from GeoDa.
  - Format file no compatible with `spatwmat`.

Generating W using Stata

We will use a geographic criterion:

- `spwmatrix`: for example 5-nn.

```
. spwmatrix gecon y_c x_c, wn(W5st) knn(5) row connect
Nearest neighbor (knn = 5) spatial weights matrix (164 x 164)
calculated successfully and the following action(s) taken:
 - Spatial weights matrix  created as Stata object(s): W5st.
 - Spatial weights matrix has been row-standardized.
Connectivity Information for the Spatial Weights Matrix
   - Sparseness: 3.049%
   - Neighbors: Min    : 5
                Mean   : 5
                Median: 5
                Max    : 5
```

It is not advisable to work with units without neighbours.
In addition, it is usual to standardize W (usually row-standardize).

Univariate spatial tests

The following statistics provide a measure of global spatial autocorrelation and allow us to know its significance.

- Moran I test (1950):

$$I = \frac{n}{S_0} \frac{\sum_i \sum_j (y_i - \overline{y}) w_{ij} (y_j - \overline{y})}{\sum_{i=1}^{N} (y_i - \overline{y})^2}.$$

- Geary c test (1954):

$$c = \frac{n-1}{2S_0} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (y_i - y_j)^2}{\sum_{i=1}^{n} (y_i - \overline{y})^2}.$$

- Getis-Ord G test (1992):

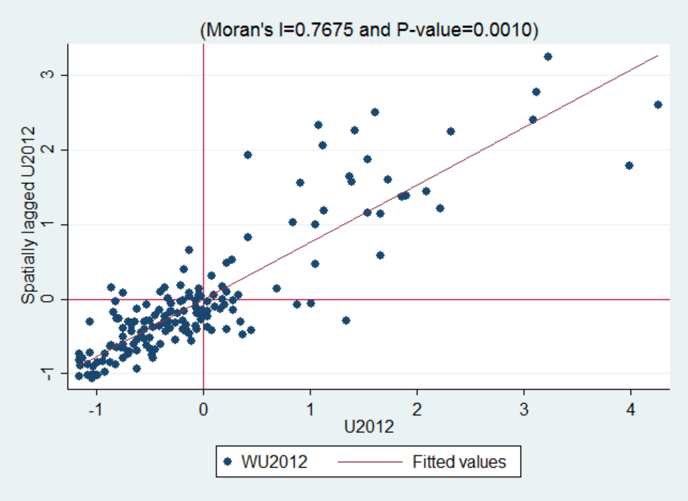
$$G = \frac{\sum_{i}^{n} \sum_{j \neq i}^{n} w_{ij} y_i y_j}{\sum_{i}^{n} \sum_{j \neq i}^{n} y_i y_j}.$$

Null hypotheses of tests: No spatial autocorrelation.

Global spatial tests in Stata

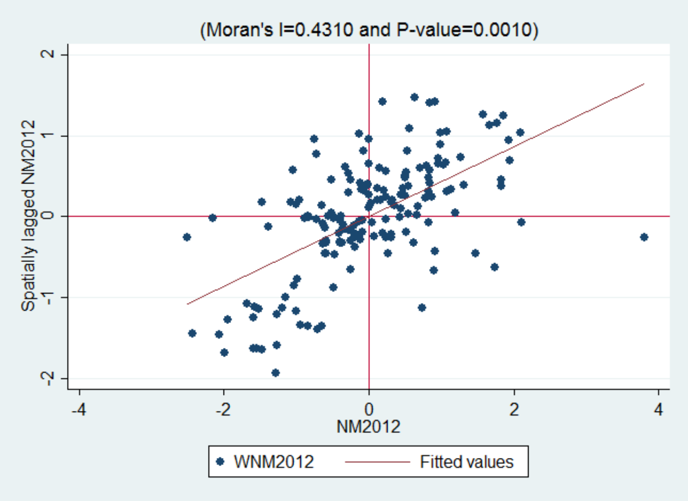
```
. spatgsa U2012, w(W5st) moran geary two
Measures of global spatial autocorrelation
---------------------------------------------------------------
Moran's I
---------------------------------------------------------------
          Variables |    I      E(I)    sd(I)      z    p-value*
--------------------+------------------------------------------
              U2012 |  0.767  -0.006   0.045  17.084   0.000
---------------------------------------------------------------
Geary's c
---------------------------------------------------------------
          Variables |    c      E(c)    sd(c)      z    p-value*
--------------------+------------------------------------------
              U2012 |  0.228   1.000   0.054 -14.282   0.000
---------------------------------------------------------------
*2-tail test

. spatgsa U2012, w(W5bin) go two
Measures of global spatial autocorrelation
---------------------------------------------------------------
Getis & Ord's G
---------------------------------------------------------------
          Variables |    G      E(G)    sd(G)      z    p-value*
--------------------+------------------------------------------
              U2012 |  0.039   0.031   0.001  11.864   0.000
---------------------------------------------------------------
*2-tail test
```

```
splagvar U2012, wname(W5st) wfrom(Stata) ind(U2012) order(1) plot(U2012)
moran(U2012)
```



Moran's I scatterplot

```
splagvar NM2012, wname(W5st) wfrom(Stata) ind(NM2012) order(1) plot(NM2012)
moran(NM2012)
```



General Index

Local indicators of spatial association

A version of Moran I test is used to detect spatial clusters in local dimension:

$$I_i(d) = \frac{(x_i - \overline{x})}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2} \sum_{j=1, j \neq i}^{n} w_{ij}(d)(x_j - \overline{x}), \qquad (3)$$

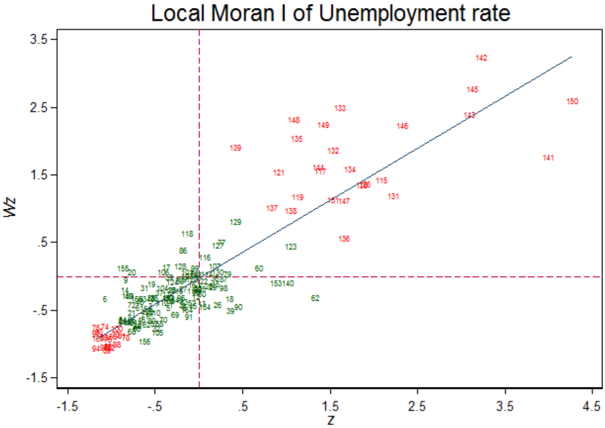
where $w_{ij}(d)$ is a weighting distance.
Null hypotheses is no spatial autocorrelation and the significance of I_i could be contrasted using normal distribution:

$$z[I_i] = \frac{[I_i - E[I_i]]}{\sqrt{\mathrm{Var}[I_i]}}.$$

This test allows grouping observations in 4 categories (see scatter Moran): High-High (H-H), Low-Low (L-L), Low-High (L-H) and High-Low (H-L).

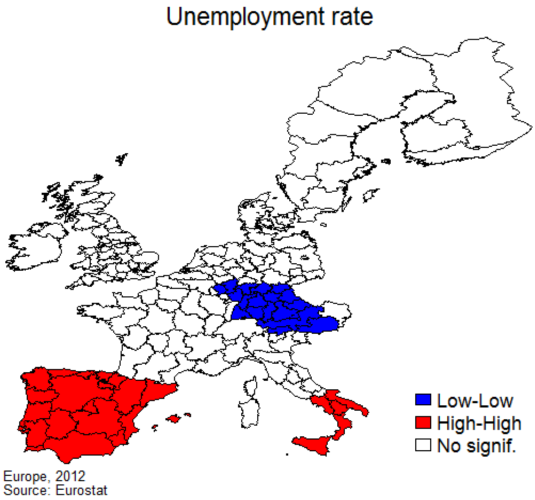
Local Moran's I scatterplot

```
genmsp_v0 U2012, w(W5st)
graph twoway (scatter Wstd_U2012 std_U2012 if pval_U2012>=0.05, msymbol(i) mlabel (id)
mlabsize(*0.6) mlabpos(c)) (scatter Wstd_U2012 std_U2012 if pval_U2012<0.05, msymbol(i) mlabel
(id) mlabsize(*0.6) mlabpos(c) mlabcol(red)) (lfit Wstd_U2012 std_U2012), yline(0, lpattern(--))
xline(0, lpattern(--)) xlabel(-1.5(1)4.5, labsize(*0.8)) xtitle("{it:z}") ylabel(-1.5(1)3.5, angle(0)
labsize(*0.8)) ytitle("{it:Wz}") legend(off) scheme(s1color) title("Local Moran I of Unemployment
rate") splagvar NM2012, wname(W5st) wfrom(Stata) ind(NM2012) order(1) plot(NM2012)
moran(NM2012)
```



Local Moran's I map

```
spmap msp_U2012 using coord, id(id) clmethod(unique) title("Unemployment rate")
legend(size(medium) position(4)) ndl("No signif.") fcolor(blue red) note("Europe,
2012" "Source: Eurostat")
```



General Index

Alternatives of specification

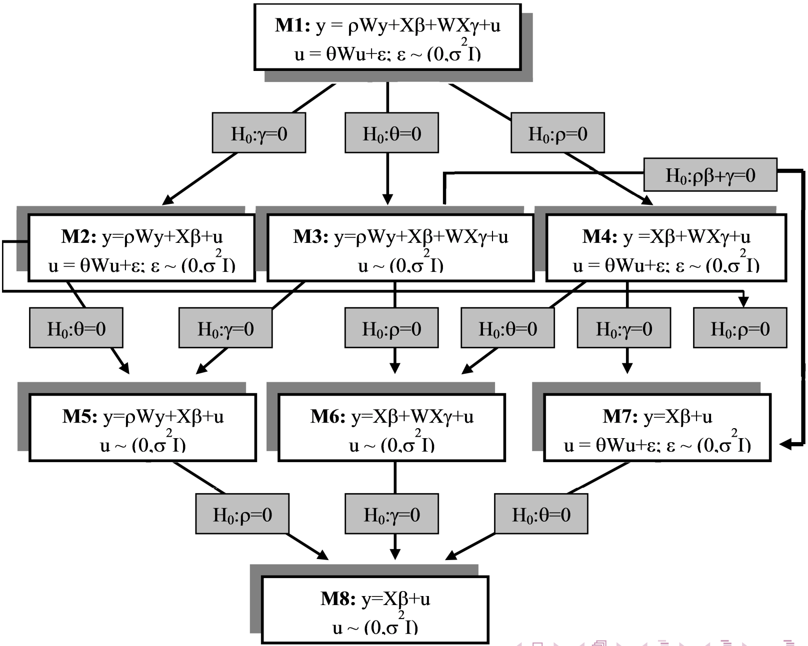
General Cliff-Ord model (Manski model)

$$
\begin{aligned}
y &= \lambda Wy + X\beta + WX\gamma + u, \\
u &= \rho Wu + \varepsilon.
\end{aligned}
$$

Imposing restrictions in γ, ρ and λ we can obtain the following models:

- $\gamma = 0, \rho = 0, \lambda \neq 0 \rightarrow SLM$.
- $\gamma = 0, \rho \neq 0, \lambda = 0 \rightarrow SEM$.
- $\gamma = 0, \rho \neq 0, \lambda \neq 0 \rightarrow SARAR$.
- $\gamma \neq 0, \rho = 0, \lambda \neq 0 \rightarrow SDM$.

Alternatives of specification



General Index

Residual tests

The first step (under the specific to general strategy M8) is to estimate a regression model and obtain the residuals.
In our case our initial model is

$$U2012 = \beta_1 + \beta_2 NM2012 + u.$$

This equation is estimated under OLS:

```
. reg U2012 NM2012
      Source |       SS       df       MS              Number of obs =     164
-------------+------------------------------           F(  1,   162) =   56.32
       Model |  1453.94714     1  1453.94714           Prob > F      =  0.0000
    Residual |  4182.20231   162  25.8160636           R-squared     =  0.2580
-------------+------------------------------           Adj R-squared =  0.2534
       Total |  5636.14945   163   34.577604           Root MSE      =   5.081

------------------------------------------------------------------------------
       U2012 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      NM2012 |  -.7011928   .0934347    -7.50   0.000    -.8856998   -.5166859
       _cons |   11.43504   .4697136    24.34   0.000     10.50749    12.36259
------------------------------------------------------------------------------
```

Residual tests

There are a set of tests that allow the detection of spatial autocorrelation:

Null hypotheses	Parameters in H_1		Test
	Spatial lag	Error lag	
$\rho = 0$	-	yes	LM_{ERROR}
	yes	yes	LM^{*}_{ERROR}
$\lambda = 0$	yes	-	LM_{LAG}
	yes	yes	LM^{*}_{LAG}
No spatial autocorrelation			Moran´s I

Spatial residual test in Stata

```
reg U2012 NM2012
spatdiag, weights(W5st)
```

```
Diagnostic tests for spatial dependence in OLS regression
------------------------------------------------------------
Diagnostics
------------------------------------------------------------
Test                            |  Statistic    df   p-value
--------------------------------+---------------------------
Spatial error:                  |
  Moran's I                     |    12.703      1     0.000
  Lagrange multiplier           |   148.081      1     0.000
  Robust Lagrange multiplier    |     0.750      1     0.386
                                |
Spatial lag:                    |
  Lagrange multiplier           |   182.220      1     0.000
  Robust Lagrange multiplier    |    34.889      1     0.000
------------------------------------------------------------
```

Conclusion of spatial residual tests

Our first step was a simple OLS:

$$U20012 = \beta_1 + \beta_2 NM2012 + u.$$

Now, according to the evidence reflected in the tests, an SLM should be estimated:

$$U20012 \quad = \lambda \left(W \times U20012 \right) + \quad \beta_1 + \beta_2 NM2012 + u.$$

Problem: How do we estimate this model?

Estimation methods from spatial models

The estimation of models with spatial components can not be performed by OLS (except simple cases such as SLX: WX). The estimation needs alternative methods: ML or IV / GMM.

Syntax:

```
spreg estimator depvar [indepvars] [if] [in],
id(varname) [options]
```

Option `estimator` allows:

- ml: maximum likelihood (ML).
- gs2sls: generalized spatial two-stage least squares (IV/GMM).

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General Index

Maximum Likelihood: spreg ml

SLM

spreg ml U2012 NM2012, id(id) dlmat(W5_st)

SEM

spreg ml U2012 NM2012, id(id) elmat(W5_st)

SARAR

spreg ml U2012 NM2012, id(id) dlmat(W5_st) elmat(W5_st)

SDM

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st)

Cliff-Ord

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st) elmat(W5_st)

Maximum Likelihood: spreg ml

SLM

spreg ml U2012 NM2012, id(id) dlmat(W5_st)

SEM

spreg ml U2012 NM2012, id(id) elmat(W5_st)

SARAR

spreg ml U2012 NM2012, id(id) dlmat(W5_st) elmat(W5_st)

SDM

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st)

Cliff-Ord

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st) elmat(W5_st)

Maximum Likelihood: spreg ml

SLM

spreg ml U2012 NM2012, id(id) dlmat(W5_st)

SEM

spreg ml U2012 NM2012, id(id) elmat(W5_st)

SARAR

spreg ml U2012 NM2012, id(id) dlmat(W5_st) elmat(W5_st)

SDM

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st)

Cliff-Ord

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st) elmat(W5_st)

Maximum Likelihood: spreg ml

SLM

spreg ml U2012 NM2012, id(id) dlmat(W5_st)

SEM

spreg ml U2012 NM2012, id(id) elmat(W5_st)

SARAR

spreg ml U2012 NM2012, id(id) dlmat(W5_st) elmat(W5_st)

SDM

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st)

Cliff-Ord

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st) elmat(W5_st)

Maximum Likelihood: spreg ml

SLM

spreg ml U2012 NM2012, id(id) dlmat(W5_st)

SEM

spreg ml U2012 NM2012, id(id) elmat(W5_st)

SARAR

spreg ml U2012 NM2012, id(id) dlmat(W5_st) elmat(W5_st)

SDM

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st)

Cliff-Ord

spreg ml U2012 NM2012 wx_NM2012, id(id) dlmat(W5_st) elmat(W5_st)

Maximum Likelihood: spreg ml

Variable	SLM	SEM	SARAR	SDM	CLIFF-ORD
$NM2012$	-0.19^{**}	-0.15^{**}	-0.16^{**}	-0.16^{**}	-0.15^{**}
$W \times NM2012$				-0.10	-0.03
$const$	2.41^{**}	10.97^{**}	1.74^{**}	2.82^{**}	1.87^{**}
$\widehat{\lambda}$	0.82^{**}		0.88^{**}	0.80^{**}	0.87^{**}
$\widehat{\rho}$		0.86^{**}	-0.33		-0.31

Nota: $^{**}p < 0.05$.

What is the best model?

Maximum Likelihood: selecting the best model

From specific to general strategy:

- Using LM tests: spatial lag model (SLM).

  But there are other posible models: SDM, SARAR and Cliff-Ord.
- Between SDM and SEM: LR_{COMFAC}.

From general to specific strategy:

- Start using Cliff-Ord model (M1) and to eliminate sequentially non-significant variables.

Maximum Likelihood: Common factor test

Assuming the *SDM* model has been estimated:

$$y = \lambda Wy + X\beta + WX\gamma + u.$$

The null and alternative hypotheses are: $\begin{array}{ll} H_0: & \gamma + \lambda\beta = 0, \\ H_1: & \gamma + \lambda\beta \neq 0. \end{array}$

Under H_0, $\gamma = -\lambda\beta$, and replacing into the *SDM* model:

$$\begin{aligned} y &= \lambda Wy + X\beta + WX(-\lambda\beta) + u = \lambda Wy + X\beta - \lambda WX\beta + u, \\ (I - \lambda W)y &= (I - \lambda W)X\beta + u. \end{aligned}$$

The last expression is summarized in *SEM*: $y = X\beta + (I - \rho W)^{-1}\varepsilon$,
where λ has been replaced by ρ.
Under null hypothesis, we have an *SEM* and, under alternative hypothesis, an *SDM*:

$$LR_{COMFAC} = 2\left[l_{|H_1} - l_{|H_0} \right] \underset{as}{\sim} \chi^2_q$$

Maximum Likelihood: Alternative models

Variable	SLM	SEM	SARAR	SDM	CLIFF-ORD
$NM2012$	-0.19^{**}	-0.15^{**}	-0.16^{**}	-0.16^{**}	-0.15^{**}
$W \times NM2012$				-0.10	-0.03
$const$	2.41^{**}	10.97^{**}	1.74^{**}	2.82^{**}	1.87^{**}
$\widehat{\lambda}$	0.82^{**}		0.88^{**}	0.80^{**}	0.87^{**}
$\widehat{\rho}$		0.86^{**}	-0.33		-0.31
LR_{COMFAC}				6.81^{**}	

Nota: $^{**}p < 0.05$.

The final model is the SLM.

IV-GMM: spivreg command

SLM

spivreg U2012 NM2012, dl(W5_st) id(id)

SEM

spivreg U2012 NM2012, el(W5_st) id(id)

SARAR

spivreg U2012 NM2012, dl(W5_st) el(W5_st) id(id)

SDM

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) id(id)

Cliff-Ord

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) el(W5_st) id(id)

IV-GMM: spivreg command

SLM

spivreg U2012 NM2012, dl(W5_st) id(id)

SEM

spivreg U2012 NM2012, el(W5_st) id(id)

SARAR

spivreg U2012 NM2012, dl(W5_st) el(W5_st) id(id)

SDM

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) id(id)

Cliff-Ord

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) el(W5_st) id(id)

IV-GMM: spivreg command

SLM

spivreg U2012 NM2012, dl(W5_st) id(id)

SEM

spivreg U2012 NM2012, el(W5_st) id(id)

SARAR

spivreg U2012 NM2012, dl(W5_st) el(W5_st) id(id)

SDM

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) id(id)

Cliff-Ord

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) el(W5_st) id(id)

IV-GMM: spivreg command

SLM

spivreg U2012 NM2012, dl(W5_st) id(id)

SEM

spivreg U2012 NM2012, el(W5_st) id(id)

SARAR

spivreg U2012 NM2012, dl(W5_st) el(W5_st) id(id)

SDM

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) id(id)

Cliff-Ord

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) el(W5_st) id(id)

IV-GMM: spivreg command

SLM

spivreg U2012 NM2012, dl(W5_st) id(id)

SEM

spivreg U2012 NM2012, el(W5_st) id(id)

SARAR

spivreg U2012 NM2012, dl(W5_st) el(W5_st) id(id)

SDM

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) id(id)

Cliff-Ord

spivreg U2012 NM2012 wx_NM2012, dl(W5_st) el(W5_st) id(id)

IV-GMM: Alternative models

Variable	SLM	SEM	SARAR	SDM	CLIFF-ORD
$NM2012$	-0.14^{**}	-0.20^{**}	-0.15^{**}	-0.14^{**}	-0.16^{**}
$W \times NM2012$				0.02	-0.06
const	1.57^{**}	10.50^{**}	1.56^{**}	1.40^{**}	2.16^{**}
$\widehat{\lambda}$	0.89^{**}		0.90^{**}	0.91^{**}	0.85^{**}
$\widehat{\rho}$		0.83^{**}	-0.27		-0.09

Nota: $^{**}p < 0.05$.

Correction by heteroskedasticity

Variable	SLM	SEM	SARAR	SDM	CLIFF-ORD
$NM2012$	-0.14^{**}	-0.21^{**}	-0.15^{**}	-0.14^{**}	-0.16^{**}
$W \times NM2012$				0.02	-0.06
$const$	1.57	10.50^{**}	1.57^{**}	1.40	2.18
$\widehat{\lambda}$	0.89^{**}		0.90^{**}	0.91^{**}	0.85^{**}
$\widehat{\rho}$		0.82^{**}	-0.18		0.02

Nota: $^{**}p < 0.05$.

Interpretation of estimated parameters

- In SLM, SARAR or SDM models, a change of the variable x_k in region i will affect the region itself and affects potentially the other regions indirectly through the spatial multiplier mechanism $((I - \lambda W)^{-1})$.
- In a linear model, the marginal effect is:

$$\frac{\partial E(y_i)}{\partial x_{ik}} = \widehat{\beta}_k \quad \frac{\partial E(y_j)}{\partial x_{ik}} = 0$$

but in spatial models with Wy and/or Wx, the second effect is not zero.

SLM. Direct and indirect effects

The marginal effect of the explanatory variable x_k on the dependent variable is:

$$\begin{aligned}
\begin{bmatrix} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{bmatrix} &= \begin{bmatrix} \frac{\partial y_1}{\partial x_{1k}} & \cdots & \frac{\partial y_1}{\partial x_{nk}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_n}{\partial x_{1k}} & \cdots & \frac{\partial y_n}{\partial x_{1k}} \end{bmatrix}, \\
&= (I_n - \lambda W)^{-1} \begin{bmatrix} \beta_k & 0 & \cdots & 0 \\ 0 & \beta_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_k \end{bmatrix}, \\
&= (I_n - \lambda W)^{-1} [\beta_k I_n], \qquad (4)
\end{aligned}$$

Direct effect: average of the elements of principal diagonal of $(I_n - \lambda W)^{-1} [\beta_k I_n]$.

Indirect effect: (spatial spillover) average of sum of rows, without of elements of principal diagonal of $(I_n - \lambda W)^{-1} [\beta_k I_n]$.

Example under SLM in Stata

If we apply the above expression to our example (SLM using MLE):

```
. mata:
--------------------- mata (type end to exit)------------------
: b = st_matrix("e(b)")
: b
                 1              2              3              4
    +-------------------------------------------------------------+
  1 |   -.1898462498     2.40790211    .8174714987    8.182474325 |
    +-------------------------------------------------------------+
: lambda = b[1,3]
: lambda
.8174714987
: S = luinv(I(rows(W))-lambda*W)
: end
----------------------------------------------------------------
. * Total effects
. mata: (b[1,1]/rows(W))*sum(S)
  -1.040090991
 * Direct effects
. mata: (b[1,1]/rows(W))*trace(S)
  -.2414164099
. * Indirect effects (spatial spillovers)
. mata: (b[1,1]/rows(W))*sum(S) - (b[1,1]/rows(W))*trace(S)
  -.7986745812
```

General Index

Fixed effect and random effects

Consider a linear model with k independent variables x_{it}:

$$y_{it} = x_{it}\beta + u_{it}, \quad (5)$$

where $i = 1, \ldots, n$, $t = 1, \ldots, T$ and u_{it} is a random error term.
This model doesn't control by heterogeneity: specific temporal variables could be affect on dependent variable.

- Solution:

$$u_{it} = \mu_i + \phi_t + \varepsilon_{it},$$

where μ_i is a common region-specific effect and ϕ_t is a common time-specific effect for all regions.
These effects could be treated as fixed or random.

Fixed or random effects

Hausman test (1978) is computed as:

$$H = (\beta_{fe} - \beta_{re})^{'} (V_{fe} - V_{re})^{-1} (\beta_{fe} - \beta_{re}),$$

where β_{fe} is the vector of coefficients of the consistent estimator `fe`, β_{re} is the vector of coefficients of the efficient estimator `re`, with V_{fe} and V_{re} as the variance-covariance matrix of `fe` and `re`, respectively. This statistic is distributed as χ^2_q, with q degrees (number of common coefficients in both models).
Hausman test can be consider as a statistic of validation of `re` estimator, null hypotheses.

To incorporate spatial effects we must have some evidence of their presence. A possible test that can be used is CD test (Pesaran, 2004):

$$CD = \sqrt{\frac{2T}{n(n-1)}} \left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \widehat{\rho}_{ij} \right),$$

where $\widehat{\rho}_{ij}$ is the correlation coefficient in the residuals between i and j:

$$\widehat{\rho}_{ij} = \widehat{\rho}_{ji} = \frac{\sum_{t=1}^{T} \widehat{u}_{it} \widehat{u}_{jt}}{\left(\sum_{t=1}^{T} \widehat{u}_{it}^2 \right)^{1/2} \left(\sum_{t=1}^{T} \widehat{u}_{jt}^2 \right)^{1/2}}, (1) \qquad (6)$$

Null hypothesis: no autocorrelation in cross-section dimension.
In Stata:

```
. xtreg U NM, fe
  (ommitted product)
. xtcsd, pes abs

 Pesaran's test of cross sectional independence =     60.169, Pr = 0.0000
 Average absolute value of the off-diagonal elements =     0.464
```

General Index

Spatial lag model

The SLM with fixed effects is:

$$\begin{gathered} y_t = \rho W y_t + X_t \beta + \mu + \varepsilon_t, \\ \varepsilon_t \sim \mathcal{N}\left[0, \sigma_\varepsilon^2 I_n\right], \end{gathered} \tag{7}$$

where

$$y_t = \begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{pmatrix}, X_t = \begin{pmatrix} x_{11t} & x_{21t} & \cdots & x_{k1t} \\ x_{12t} & x_{22t} & \cdots & x_{k2t} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1nt} & x_{2nt} & \cdots & x_{knt} \end{pmatrix}, \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix}.$$

Under random effects, this model can be written as:

$$\begin{gathered} y_t = \rho W y_t + X_t \beta + \underbrace{\mu + \varepsilon_t}_{u_t}, \\ \varepsilon_t \sim \mathcal{N}\left[0, \sigma_\varepsilon^2 I_n\right], \mu \sim \mathcal{N}\left[0, \sigma_\mu^2 I_n\right]. \end{gathered} \tag{8}$$

SLM. Direct and indirect effects

The partial effect of one unit increases on the SLM model is as follows:

$$
\begin{aligned}
\begin{bmatrix} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{bmatrix} &= \begin{bmatrix} \frac{\partial y_1}{\partial x_{1k}} & \cdots & \frac{\partial y_1}{\partial x_{nk}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_n}{\partial x_{1k}} & \cdots & \frac{\partial y_n}{\partial x_{1k}} \end{bmatrix}, \\
&= (I_n - \rho W)^{-1} \begin{bmatrix} \beta_k & 0 & \cdots & 0 \\ 0 & \beta_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_k \end{bmatrix}, \\
&= (I_n - \rho W)^{-1} [\beta_k I_n], \qquad (9)
\end{aligned}
$$

Direct effect: average of the elements of principal diagonal of $(I_n - \rho W)^{-1} [\beta_k I_n]$.
Indirect effect: (spatial spillover) average of sum of rows, without of elements of principal diagonal of $(I_n - \rho W)^{-1} [\beta_k I_n]$.

Spatial Error Model

The *SEM* model with fixed effects is:

$$y_t = X_t\beta + \mu + \varepsilon_t$$

$$\varepsilon_t = \rho W \varepsilon_t + \eta_t \tag{10}$$

$$\eta_t \sim \mathcal{N}\left[0, \sigma_\eta^2 I_n\right]$$

and the version of *SEM* model with fixed effects is:

$$y_t = X_t\beta + \underbrace{\mu + \varepsilon_t}_{u_t},$$
$$\varepsilon_t = \rho W \varepsilon_t + \eta_t,$$
$$\eta_t \sim \mathcal{N}\left[0, \sigma_\eta^2 I_n\right],\ \mu \sim \mathcal{N}\left[0, \sigma_\mu^2 I_n\right],$$

Spatial Durbin Model

SDM specification:

$$y_t = \rho W y_t + X_t \beta + W X_t \gamma + \varepsilon_t, \tag{11}$$

with direct-indirect effects:

$$\begin{aligned}
\begin{bmatrix} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{bmatrix} &= \begin{bmatrix} \frac{\partial y_1}{\partial x_{1k}} & \cdots & \frac{\partial y_1}{\partial x_{nk}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_n}{\partial x_{1k}} & \cdots & \frac{\partial y_n}{\partial x_{1k}} \end{bmatrix}, \\
&= (I_n - \rho W)^{-1} \begin{bmatrix} \beta_k & w_{12}\gamma_k & \cdots & w_{1n}\gamma_k \\ w_{21}\gamma_k & \beta_k & \cdots & w_{2n}\gamma_k \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1}\gamma_k & w_{n2}\gamma_k & \cdots & \beta_k \end{bmatrix}, \\
&= (I_n - \rho W)^{-1} [\beta_k I_n + \gamma_k W], \qquad (12)
\end{aligned}$$

command xsmle

SLM

```
xsmle U NM, fe wmat(W5_st) mod(sar) hausman
```

SEM

```
xsmle U NM, fe emat(W5_st) mod(sem) hausman
```

SARAR

```
xsmle U NM, fe wmat(W5_st) emat(W5_st) vce(r) mod(sac)
```

SDM

```
xsmle U NM, fe wmat(W5_st) mod(sdm) hausman
```

command xsmle

SLM

```
xsmle U NM, fe wmat(W5_st) mod(sar) hausman
```

SEM

```
xsmle U NM, fe emat(W5_st) mod(sem) hausman
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SARAR

```
xsmle U NM, fe wmat(W5_st) emat(W5_st) vce(r) mod(sac)
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SDM

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xsmle U NM, fe wmat(W5_st) mod(sdm) hausman
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command xsmle

SLM

```
xsmle U NM, fe wmat(W5_st) mod(sar) hausman
```

SEM

```
xsmle U NM, fe emat(W5_st) mod(sem) hausman
```

SARAR

```
xsmle U NM, fe wmat(W5_st) emat(W5_st) vce(r) mod(sac)
```

SDM

```
xsmle U NM, fe wmat(W5_st) mod(sdm) hausman
```

command xsmle

SLM

```
xsmle U NM, fe wmat(W5_st) mod(sar) hausman
```

SEM

```
xsmle U NM, fe emat(W5_st) mod(sem) hausman
```

SARAR

```
xsmle U NM, fe wmat(W5_st) emat(W5_st) vce(r) mod(sac)
```

SDM

```
xsmle U NM, fe wmat(W5_st) mod(sdm) hausman
```

Alternative Models

Variable	SLM	SEM	SARAR	SDM
NM	-0.16^{**}	-0.16^{**}	-0.13^{**}	-0.15^{**}
$W \times NM$				-0.02
$\widehat{\rho}$	0.79^{**}		0.84^{**}	0.78^{**}
$\widehat{\lambda}$		0.87^{**}	-0.29^{**}	
COMFAC				88.41^{**}
Spatial effects (long run)				
Directs	-0.19^{**}		-0.17^{**}	-0.18^{**}
Indirects	-0.54^{**}		-0.66^{**}	-0.58^{**}
Totals	-0.74^{**}		-0.84^{**}	-0.76^{**}
AIC	2352.79	2733.18	2629.62	2642.88

Nota: $^{**}p < 0.05$.

General Index

Dynamic SLM models

Dynamic versions of Spatial Lag Model (*dynSLM*):

$$y_t = \tau y_{t-1} + \rho W y_t + X_t \beta + \mu + \varepsilon_t, \tag{13}$$

$$y_t = \psi W y_{t-1} + \rho W y_t + X_t \beta + \mu + \varepsilon_t, \tag{14}$$

$$y_y = \tau y_{t-1} + \psi W y_{t-1} + \rho W y_t + X_t \beta + \mu + \varepsilon_t, \tag{15}$$

These model give us the option to obtain direct and indirect effects in short and long run:
Short run (assuming $\tau = \psi = 0,$):

$$\left[\begin{array}{ccc} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{array} \right]_t = \left(I_n - \rho W\right)^{-1} \left[\beta_k I_n\right]. \tag{16}$$

Long run (assuming $y_t = y_{t-1} = y^*$):

$$\left[\begin{array}{ccc} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{array} \right]_t = \left[\left(1-\tau\right) I_n - \left(\rho + \psi\right) W\right]^{-1} \left[\beta_k I_n\right]. \tag{17}$$

Dynamic SDM models

Dynamic versions of Spatial Durbin Model (*dynSDM*):

$$y_t = \tau y_{t-1} + \rho W y_t + X_t \beta + W X_t \gamma + \mu + \varepsilon_t, \quad (18)$$

$$y_t = \psi W y_{t-1} + \rho W y_t + X_t \beta + W X_t \gamma + \mu + \varepsilon_t, \quad (19)$$

$$y_t = \tau y_{t-1} + \psi W y_{t-1} + \rho W y_t + X_t \beta + W X_t \gamma + \mu + \varepsilon_t, \quad (20)$$

These model give us the option to obtain direct and indirect effects in short and long run:
Short run (assuming $\tau = \psi = 0,$):

$$\left[\begin{array}{ccc} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{array} \right]_t = (I_n - \rho W)^{-1} [\beta_k I_n + \gamma_k W] . \quad (21)$$

Long run (assuming $y_t = y_{t-1} = y^*$):

$$\left[\begin{array}{ccc} \frac{\partial y}{\partial x_{1k}} & \cdots & \frac{\partial y}{\partial x_{nk}} \end{array} \right]_t = [(1-\tau) I_n - (\rho + \psi) W]^{-1} [\beta_k I_n + \gamma_k W] . \quad (22)$$

Example of unemployment-migration. Serial correlation

Wooldridge (2002) develops a simple statistic for autocorrelation detection.
Drukker (2003) implements it in Stata, `xtserial` command:

```
. xtserial U NM
Wooldridge test for autocorrelation in panel data
H0: no first-order autocorrelation
    F(  1,      163) =    110.869
           Prob > F =      0.0000
```

xsmle command

SLM 1 (eq. 13)

```
xsmle U NM, dlag(1) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

SLM 2 (eq. 14)

```
xsmle U NM, dlag(2) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

SLM 3 (eq. 15)

```
xsmle U NM, dlag(1) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

xsmle command

SLM 1 (eq. 13)

```
xsmle U NM, dlag(1) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

SLM 2 (eq. 14)

```
xsmle U NM, dlag(2) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

SLM 3 (eq. 15)

```
xsmle U NM, dlag(1) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

xsmle command

SLM 1 (eq. 13)

```
xsmle U NM, dlag(1) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

SLM 2 (eq. 14)

```
xsmle U NM, dlag(2) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

SLM 3 (eq. 15)

```
xsmle U NM, dlag(1) fe wmat(W5_st) type(ind) mod(sar)
effects nsim(499)
```

Alternative models of dynamic SLM

Variable	SLM 1	SLM 2	SLM 3
$U(t-1)$	0.46^{**}		0.75^{**}
NM	0.07^{**}	-0.03	0.10^{**}
$W \times U$	0.75^{**}	0.76^{**}	0.99^{**}
$W \times U(t-1)$		0.30^{**}	-0.51^{**}
Spatial effects (short run)			
Directs	0.08^{**}	-0.03	0.15
Indirects	0.19^{**}	-0.08	-12.52
Totals	0.27^{**}	-0.11	-12.37
Spatial effects (long run)			
Directs	0.10	0.81	0.22
Indirects	-0.44^{**}	-1.25	-0.66
Totals	-0.34^{**}	-0.45	-0.44^{**}
AIC	1839.55	1972.80	1812.61

Nota: $^{**}p < 0.05$.

Summing up

- Stata has incorporated tools for spatial analysis.
- ESDA can be carried out completely, as in others software.
- Also, for cross-section data, the most common spatial specifications can be estimated by ML and/or IV/GMM.
- For panel data, recent developments provide alternatives for estimating static and dynamic models.
- Main results of the impact of net migration:
  - Cross-section model: SLM shows a negative impact in unemployment (long run effect).
  - Panel-data: Dynamic SLM shows a positive impact in short run (orthodox theory) and negative impact in long run, total effect (NEG theory).

Summing up

- Stata has incorporated tools for spatial analysis.
- ESDA can be carried out completely, as in others software.
- Also, for cross-section data, the most common spatial specifications can be estimated by ML and/or IV/GMM.
- For panel data, recent developments provide alternatives for estimating static and dynamic models.

- Main results of the impact of net migration:
  - Cross-section model: SLM shows a negative impact in unemployment (long run effect).
  - Panel-data: Dynamic SLM shows a positive impact in short run (orthodox theory) and negative impact in long run, total effect (NEG theory).

Some references

Theoretical references:

- Anselin, L. and A. Bera (1998). "Spatial dependence in linear regression models with an Introduction to Spatial Econometrics," Handbook of Applied Economic Statistics, pp. 237-289.
- Brueckner, J. (2003). "Strategic interaction among governments: An overview of empirical studies," International Regional Science Review, 26(2).
- Elhorst, J. P. (2013). "Spatial Panel Models," Chapter 82, en Fischer y Nijkamp (eds.) Handbook of Regional Science. Heidelberg: Springer.

Applied references:

- Ertur, C. and W. Koch (2007). "Growth, technological interdependence and spatial externalities: Theory and evidence," Journal of Applied Econometrics, 22.
- Le Gallo, J. and C. Ertur (2003). "Exploratory spatial data analysis of the distribution of regional per capita GDP in Europe, 1980– 1995," Papers in regional science, 82(2).
- Fingleton, B. (2006). "A cross-sectional analysis of residential property prices: the effects of income, commuting, schooling, the housing stock and spatial interaction in the English regions," Papers in Regional Science, 85(3).

Some references

Books:

- Anselin, L. (1988). Spatial Econometrics: Methods and Models.
- LeSage, J. and Pace (2009). An introduction to spatial econometrics.
- Elhorst, J. P. (2014). Spatial econometrics: from cross-sectional data to spatial panels.

Stata:

- Belotti, F. et al (2016). XSMLE: Stata module for spatial panel data models estimation. Statistical Software Components.
- Drukker, D. M. et al. (2011). A command for estimating spatial-autoregressive models with spatial-autoregressive disturbances and additional endogenous variables. Econometric Reviews, 32, 686-733.
- Drukker, D. M. et al. (2013). Creating and managing spatial-weighting matrices with the spmat command. Stata Journal, 13(2), 242-286.
- Pisati, M. (2008). SPMAP: Stata module to visualize spatial data. Statistical Software Components.