Stocks and Noise:
Representation of the Evolution of Unstable Economies

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- The capital held in a national economy can be seen, at a very high level, as a *stock*, i.e. a homogeneous good that yields a flow of benefits in time.
- Assuming that the economy is *closed*, benefits can be devoted either to consumption or to acquire more of the stock.
- This analogy might help to understand why certain countries, that were expected to accumulate wealth and become affluent, ended up stagnating.
- An important explanation for this phenomenon focuses on the role of institutions in economic growth. In the stock analogy this feature could be captured by the prevalence of noise affecting both the returns and the value of future outcomes.

- The economy considered here consists of a single production unit.
- There is a single representative agent in the economy with the following prospective utility function:

$$W({}_0c) = \sum_{s=0}^{\infty} \left\{ \prod_{t=0}^{s-1} \alpha(c_t) \right\} u(c_s)$$

where $u(c_s)$ is the agent's instantaneous utility of consuming c_s at s, and the real-valued function $\alpha(c_t)$ is the psychological factor of time preference.

- $\alpha(\cdot)$ is increasing in consumption (and therefore in income).

The prospective utility function and the psychological time preference function have the following properties:

1. $u(c)$ and $\alpha(c)$ are continuous on R_+ and twice differentiable for $c > 0$.

2. $u' > 0 > u''$, and $\lim_{c\to 0+} u'(c) = +\infty$, $\lim_{c\to\infty} u'(c) = 0$, $u(0) \geq 0$.

3. $\alpha' > 0 > \alpha''$, $\alpha(0) > 0$.

4. $0 < \alpha(c) \leq \bar{\alpha} < 1$ for some constant $\bar{\alpha}$, for all $c \geq 0$.

5. $|\alpha'' u + \alpha u''| \geq 2|\alpha' u'|$.

This prospective utility allows us to define an (individual) welfare path $\{W_t\}$ such that:

$$W_t \equiv W[{}_t c] = \sum_{s=t}^{\infty} \left\{ \prod_{v=t}^{s-1} \alpha(c_v) \right\} u(c_s)$$

where ${}_t c$ is the consumption path beginning at period t. The sequence $\{W_t\}$ satisfies the difference equation:

$$W_t = u(c_t) + \alpha(c_t) W_{t+1}.$$

The second member, $V(c, W) \equiv u(c) + \alpha(c)W$, is called an *utility aggregator*. It has been proved that, with the properties assumed here, this function is continuous, increasing in its arguments and satisfies the Lipschitz condition of order one. This shows that successive approximations lead to a single value of the prospective utility.

The technology consists in a simple neoclassical aggregate production function: a real-valued production function $f(k)$, where k is the positive per capita capital. In turn, $f(k)$ has the following properties:

1. It is continuous, twice continuously differentiable for $k > 0$.
2. $f(0) = 0$, $f^{'} > 0$, $f^{''} < 0$, $lim_{k \to 0^+} f^{'}(k) = +\infty$

A final element in the description of the economy is a random variable $\theta(c) \in [0, 1]$ representing the *institutional noise* affecting the economy.

- For each c_t, $1 - \theta(c_t)$ indicates the proportion of the wealth produced in $t + 1$ that is lost due to the malfunctioning of the institutions in the economy.
- The distribution of $\theta(c)$ stochastically dominates $\theta(c')$ for $c > c'$.
- That is, the higher the well-being in the previous period (roughly represented by the amount of consumption) the lower will tend to be the waste of resources in the next one.

- There exists only one good, used both for consumption and for accumulation.
- $f(\cdot)$ is net of depreciation and of maintenance costs.
- The labor force is assumed constant, and all relevant variables are expressed in *per capita* terms.
- A capital path ${}_0k$ is admissible and feasible for an initial capital stock k if $k_0 = k$ and for $0 \leq t$:

$$0 \leq k_{t+1} \leq \theta(c_t) f(k_t)$$

The optimization problem faced the representative agent is:

$$v(k_0) = \ \text{Max}_{(c_0, c_1, \ldots)} \sum_{t=0}^{\infty} \beta_t u(c_t)$$

s.t.

$$k_{t+1} \leq \theta(c_t) f(k_t) - c_t$$

$$\beta_{t+1} \leq \alpha(c_t) \beta_t$$

$$k_0 \text{ given; } \beta_0 = 1$$

Notice that β_t actually is a shorthand for the recursively determined weight of $u(c_t)$, i.e. $\beta_t = \prod_{s=0}^{t-1} \alpha(c_s)$.

Since $k_{t+1} \leq \theta(c_t)f(k_t)$, this problem can be recast in terms of Bellman's equation:

$$v(k_t) = \text{ Max}_{c_t} \{u(c_t) + \alpha(c_t)v(\theta(c_t)f(k_t) - c_{t+1})\} \ \ (\mathbf{Eq1})$$

k_t can be seen as the amount of stock held in the economy, $\theta(c_t)f(k_t)$ its return and the sequence $k_0, k_1, \ldots, k_t, \ldots$ as the evolution of the "portfolio" consisting only of the stock.

- The price of k_t, p_{k_t}, can be found as follows. The wealth of the economy obtains as

$$W = p_{k_t} k_t \quad (*)$$

meaning that the wealth of the agent (W) is totally spent on buying the portfolio composed only by k_t.

- Wealth can be expressed in terms of the noise built in k_t:

$$W = M P_\mu + \Sigma P_\sigma + \Gamma P_\gamma + \Lambda P_\lambda + \Delta P_\delta \quad (**)$$

- Combining $(*)$ and $(**)$ we obtain

$$p_{k_t} = \frac{M}{k_t} P_\mu + \frac{\Sigma}{k_t} P_\sigma + \frac{\Gamma}{k_t} P_\gamma + \frac{\Lambda}{k_t} P_\lambda + \frac{\Delta}{k_t} \quad (\mathbf{Eq2})$$

- The dynamics of (**Eq1**) and (**Eq2**) capture the behavior of this economy.